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The University of Missouri-Rolla

Chemical Engineering Academy Lectureship

"On Classroom Teaching and Textbook Writing"

"An Example: the Mechanical Energy Balance"

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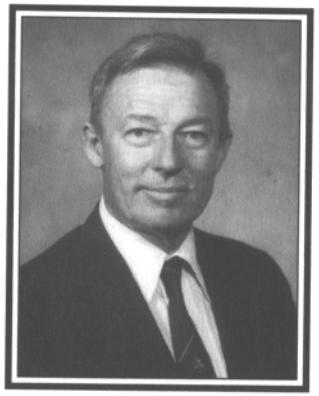

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Professor Bird retired in 1992 after teaching forty years - one at Cornell University and thirty-nine at the University of Wisconsin. He co-authored with W. E. Stewart and E. N. Lightfoot the first text for general classroom use on *Transport Phenomena*, now in its 58th printing. The English language edition has sold over 200,000 copies. He is co-author with R. C. Armstrong, O. Hassager, and C. F. Curtiss *Dynamics of Polymeric Liquids*, which appeared in a second edition in 1987. Another major work is *Molecular Theory of Gases and Liquids* with J. O. Hirschfelder and C. F. Curtiss. Professor Bird has maintained extensive contacts in both Holland and Japan. He was a Fulbright Professor at Kyoto University and Nagoya University in Japan in 1962-63. He taught twice at the Technical University of Delft, first in 1958 and again in 1994 when he was the first J. M. Burgers Professor. In the fall of 1994 he was a visiting professor at Université Catholique de Louvain in Belgium.

Professor Bird is currently collaborating with C. F. Curtiss of the Theoretical Chemistry Institute at the University of Wisconsin on further developments of the fundamentals of the kinetic theory of polymers.

Professor Bird has an abiding interest in teaching chemical engineering as evidenced by his article in Chemical Engineering Education entitled The Seven Rules for Teaching.

On Classroom Teaching and Textbook Writing

An Example: The Mechanical Energy Balance

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§1. The evolution of "engineering sciences"

In the first four decades of this century science courses were taught in science departments, and in engineering departments science was applied to the solution of engineering problems. Since World War II there has been a gradual shift of the teaching of many basic science topics (such as thermodynamics, fluid dynamics, and mechanics) to the college of engineering, where these subjects are taught under the general title of "engineering sciences." These topics appear to be of little interest to the physicists, who have turned their attention to nuclear structure, nonlinear optics, string theory, and other "frontier" topics.

This shifting of the engineering science courses into the colleges of engineering means that the engineering professors now have the responsibility for the further development of these subjects through research and for the maintenance of standards in these disciplines. However, current funding patterns favor those who are involved in the applied areas and who can show that their research contributes to societal goals [1]. Those who wish to dedicate themselves to teaching engineering-science courses and doing very fundamental research do so at their own peril.

The result is a "misalignment" between the teaching and

research duties of engineering faculty members. Professors now have vigorous research programs, often well funded, and they routinely participate in many technical meetings and workshops. They also spend a great deal of time writing grant proposals to compete for funds for their research programs. Their research areas seldom correspond to the engineering science courses that form a basic part of the undergraduate and graduate teaching program. As a result they find themselves torn between pursuit of research and research dollars on the one hand, and teaching the fundamental subjects on the other.

§2. Faculty responsibilities for teaching and book-writing

It takes a lot of time to prepare well-organized, inspiring, and responsible lectures to undergraduates. The first time that one teaches a course, it is not unreasonable for a professor to spend 10-12 hours per lecture. In subsequent years only 3-4 hours of preparation per lecture may be needed. Because of the pressures for grant-proposal writing and grant administration, most professors do not have such large blocks of time available, and therefore they must rely heavily on textbooks.

But textbook-writing requires even larger blocks of time, enormous amounts of energy, and personal sacrifices. An author must be completely up-to-date in his field, must pay scrupulous attention to detail, and must be thoroughly familiar with the published literature as well as the future directions of the subject. The writing of textbooks receives little encouragement by administrators, and many books are unfortunately written without adequate time and facilities. I have already written articles about some of these problems elsewhere and need not pursue this subject further here [2,3].

§3. The macroscopic mechanical energy balance (MMEB)

Instead it might be more instructive to take a specific example of the challenge of preparing responsible technical presentations for students and teachers. A well-known equation that can be found in many engineering textbooks and handbooks is the "(engineering) Bernoulli equation" or the "(macroscopic) mechanical energy balance," abbreviated here as "MMEB." For an engineering flow system (see Figure 1) operating at steady state, the MMEB is usually given in textbooks as:

$$\Delta \tfrac{1}{2}v^2 + g\Delta z + \int_1^2 \frac{1}{\rho}dp + \hat{W}_m + \hat{E}_v = 0 \qquad (1)$$

in which the symbols have the following meaning:

ΔX = X at the exit plane ("2") minus X at the entry plane ("1")
v = the fluid velocity of the entering or leaving stream
g = the gravitational acceleration
z = the elevation at the entry or exit port
ρ = the fluid density
p = the fluid pressure
$\hat{W}_m$ = the rate at which the system does work on the surroundings by means of moving parts (sometimes called "shaft work")
$\hat{E}_v$ = the rate at which mechanical energy is converted into heat by viscous dissipation (sometimes called, inappropriately, "lost work")
$\hat{}$ = a quantity per unit mass

This MMEB has been used for many decades by mechanical, civil, and chemical engineers to describe the relations among the various mechanical quantities in a flow system.

§4. Confusion about the origin of the MMEB

Now, what do the chemical engineering textbooks say about the *origin* of this equation? Without identifying the authors, I will quote what some of them have written about the MMEB cited above (underlines are mine):

- "An adequate appreciation of the underlying significance of [the MMEB] cannot be had except in the light of the second law."
- "It must be distinctly understood…that this [MMEB] is only a special case of the more general law of the conservation of energy."
- "For certain applications many writers have preferred to put the general energy equation...in the following mechanical energy form..."
- "Equation (12.07) is known as the [MMEB] and represents a grand transport process in which the one flux (the flow of fluid) takes place under a series of independently variable potential differences: a pressure difference, an internal energy difference, an entropy difference, a gravitational level difference and a kinetic velocity difference."

- "The reader should realize that the [MMEB] is not a new balance but rather a summary of the three momentum balances."
- "The most useful form of the energy equation is one that replaces the internal energy term in [the macroscopic energy balance] with a thermodynamic equivalent. We will require the use of some basic thermodynamics, including the notion of *entropy*."
- "The total energy balance...can be written in a form involving only mechanical energies..."
- "The first and second laws of thermodynamics may be combined to yield a macroscopic mechanical energy balance."
- The [macroscopic] mechanical energy balance equation, an alternate form of the general conservation of energy, is often in a more convenient form for problem solving...The [macroscopic] mechanical energy balance arises from a consideration of the conservation of momentum and the laws of thermodynamics."

The above comments suggest that the MMEB (a) is an alternative form of the total energy balance, or (b) is derived from the second law of thermodynamics, or (c) is a consequence of the law of conservation of momentum. In some textbooks there is no definitive statement as to the origin of Eq. (1). Clearly the above statements are in conflict with one another, and not all of them can be correct. In fact, none of them are.

Is it any wonder that students have trouble understanding the MMEB? What do chemical engineering instructors do when faced with presenting Eq. (1) to an undergraduate class? What do chemical engineering graduate students do while studying for comprehensive examinations when they find conflicting statements like these?

As a student I was totally mystified by the MMEB, but, like most students, I dutifully substituted numbers into the equation and solved the homework problems with no real understanding of the equation. As a young instructor, I was then faced with trying to clarify the situation. After weeks of searching in vain for a derivation of the MMEB, I finally resorted to deriving the equation myself [4]. However, before discussing this derivation, it is instructive to examine the derivation of the older "classical Bernoulli equation."

§5. The classical Bernoulli equation for the steady-state flow of an inviscid fluid

Over two centuries ago Daniel Bernoulli suggested that there should be some kind of relation among the various forms of energy in a flow system. In his famous treatise, *Hydrodynamica*, published in 1738 he did not derive the equation that bears his name. It was actually Leonhard Euler who first published a derivation of the *Bernoulli equation* in 1755.

We can derive this equation from the equation of motion (which is based on Newton's second law of motion, or on a momentum balance):

$$\rho\frac{D\mathbf{v}}{Dt} = -\nabla p - [\nabla\cdot\tau] - \rho\nabla\hat{\Phi} \qquad (2)$$

in which τ is the stress tensor, and $\hat{\Phi}$ is the potential energy per unit mass, related to the gravitational acceleration vector by $\mathbf{g} = -\nabla\hat{\Phi}$. We now assume that

- The fluid is inviscid, so that $\tau = 0$.
- The flow is steady, so that $\partial\mathbf{v}/\partial t = 0$.
- Gravity is acting in the minus z-direction and is independent of time, so that $\hat{\Phi} = gz$, with g being constant.

When we use the vector identity $[\mathbf{v}\cdot\nabla\mathbf{v}] = \nabla\frac{1}{2}v^2 - [\mathbf{v}\times[\nabla\times\mathbf{v}]]$, the equation of motion becomes

$$\rho\nabla\tfrac{1}{2}v^2 - \rho[\mathbf{v}\times[\nabla\times\mathbf{v}]] = -\nabla p - \rho g\nabla z \qquad (3)$$

Next we divide the equation by ρ, and then we form the dot product of Eq. (3) with the unit vector in the flow direction $\mathbf{s} = \mathbf{v}/|\mathbf{v}|$. The term containing the curl of $\mathbf{v}$ then vanishes (a nice exercise in vector analysis), and $(\mathbf{s}\cdot\nabla) = d/ds$, where s is the distance along a streamline. Thus we get

$$\frac{d}{ds}\left(\tfrac{1}{2}v^2\right) = -\frac{1}{\rho}\frac{d}{ds}p + g\frac{d}{ds}z \qquad (4)$$

When this is integrated along a streamline from point "1" to point "2," we get

$$\tfrac{1}{2}\left(v_2^2 - v_1^2\right) + \int_{p_1}^{p_2}\frac{1}{\rho}dp + g(z_2 - z_1) = 0 \qquad (5)$$

This is the (classical) *Bernoulli equation*. It interrelates the pressures, velocities, and elevations at two points on any streamline in the steady-state flow of an inviscid fluid. This derivation can be found in many textbooks [5].

This Bernoulli equation was obtained from the equation of motion, which in turn comes from a momentum balance at the microscopic level. The Bernoulli equation can, however, also be obtained from the energy equation for an inviscid, non-thermally-conducting fluid (i.e., a fluid with no viscosity or thermal conductivity), although this derivation does not seem to be generally known [6]. The classical Bernoulli equation can thus be regarded as resulting either from a momentum balance or from the first law of thermodynamics appropriately generalized for open systems.

For a viscous fluid in the flow system of Figure 1, Eq. (5) can be modified by the *ad hoc* addition of two terms: $\hat{W}_m$, to account for the work done on the surroundings via moving parts, and $\hat{E}_v$ to account for the degradation of mechanical energy into thermal energy. This "plausibility argument" then gives Eq. (1), and probably this is how the MMEB in Eq. (1) was first obtained. Of course, this method of obtaining Eq. (1) gives no basic understanding of $\hat{E}_v$ and $\hat{W}_m$ in terms of the stresses and velocity gradients in the fluid, nor does it explain how one chooses a "representative streamline" through a complex piece of equipment with moving parts.

§6. Two mathematical formulas needed for deriving the MMEB

Before proceeding we interrupt the main train of thought to give two mathematical formulas:

(a) From one-dimensional calculus we know that for a function $f(x)$

$$\int_{x_1}^{x_2}\left(\frac{d}{dx}f\right)dx = f(x_2) - f(x_1) \qquad (6)$$

This equation just says that, when the derivative of a function is integrated over an interval, the result is the difference of the values of the function at the two ends of the interval.

In three dimensions there is an analogous formula for a vector function $\mathbf{f}(x,y,z)$

$$\int_V (\nabla\cdot\mathbf{f})dV = \int_S (\mathbf{n}\cdot\mathbf{f})dS \qquad (7)$$

in which $\mathbf{n}$ is the outwardly directed unit normal vector at each element of surface dS. This states that, when one integrates the divergence of $\mathbf{f}$ over a volume in space, the result is the surface integral of the normal component of the vector $\mathbf{f}$. This is the *Gauss divergence theorem*, usually taught in sophomore calculus courses.

(b) In the one-dimensional calculus there is another famous equation:

$$\frac{d}{dt}\int_{x_1(t)}^{x_2(t)} f(x,t)dx = \int_{x_1(t)}^{x_2(t)}\frac{\partial f}{\partial t}dx + f(x_2,t)\frac{dx_2}{dt} - f(x_1,t)\frac{dx_1}{dt} \qquad (8)$$

This is the *Leibniz formula* for differentiating an integral. We are here interested in this theorem when t is the time. In taking the time derivative of the integral, we must account for the change of the integrand with respect to time and also the change in the limits of the integral with time. The quantities dx_1/dt and dx_2/dt are the velocities of the terminal points of the integration range. Equation (8) is an extraordinarily useful equation.

The three-dimensional version of this formula is less well known [Ref. 7, Eq. A.5-5; Ref 8, p. 1-51]:

$$\frac{d}{dt}\int_{V(t)} f(x,y,z,t)dV = \int_{V(t)}\frac{\partial f}{\partial t}dV + \int_{S(t)}(\mathbf{n}\cdot\mathbf{v}_S)f dS \qquad (9)$$

This equation describes the time rate of change of an integral over a volume whose shape and magnitude change with time. Here, as in Eq. (8), the time derivative of the integral accounts for the change of the integrand with respect to time and also the change in the shape of the bounding surface with time. The quantity $\mathbf{v}_S$ is the velocity of a surface element dS. A less general version of this equation is called the *Reynolds transport theorem* [9]. [*Caution*: Students should not be encouraged to use Eqs. (7) and (9) until they have actually calculated the left and right sides of these equations for several specific functions and for several specific volumes.]

§7. Derivation of the MMEB

The derivation of the classical Bernoulli equation given

above suggests that we should begin by forming the dot product of the velocity $\mathbf{v}$ with the equation of motion in Eq. (2). This gives

$$\rho\frac{D}{Dt}\left(\tfrac{1}{2}\mathbf{v}^2+\hat{\Phi}\right)=-(\mathbf{v}\cdot\nabla p)-(\mathbf{v}\cdot[\nabla\cdot\tau]) \qquad (10)$$

which is the *equation of change for mechanical energy* (cf. Eq. 3.3-1 of Ref. [7]). This can be rearranged by using the equation of continuity and some standard vector-tensor identities, to give

$$\frac{\partial}{\partial t}\left(\tfrac{1}{2}\rho v^2+\rho\hat{\Phi}\right)=-\left(\nabla\cdot\left(\tfrac{1}{2}\rho v^2+\rho\hat{\Phi}\right)\mathbf{v}\right)-(\nabla\cdot p\mathbf{v})-(\nabla\cdot[\tau\cdot\mathbf{v}]) +p(\nabla\cdot\mathbf{v})+(\tau:\nabla\mathbf{v}) \qquad (11)$$

This equation states that the time rate of change of the kinetic and potential energies at a point in the flow system results from (a) the net addition of kinetic and potential energies by convection, (b) the work done by the pressure and viscous forces, and (c) the rate of conversion of mechanical energy into thermal energy by the pressure and viscous forces.

Next we integrate Eq. (11) over the entire volume of the flow system, which is changing with time because of the moving parts in the system. In doing this, we use the Gauss divergence theorem to transform the volume integrals into surface integrals, and we also apply the Leibniz formula to the first term. This gives

$$\frac{d}{dt}\int_{V(t)}\left(\tfrac{1}{2}\rho v^2+\rho\hat{\Phi}\right)dV=-\int_{S(t)}\left(\mathbf{n}\cdot\left(\tfrac{1}{2}\rho v^2+\rho\hat{\Phi}\right)(\mathbf{v}-\mathbf{v}_S)\right)dS -\int_{S(t)}(\mathbf{n}\cdot p\mathbf{v})dS-\int_{S(t)}(\mathbf{n}\cdot[\tau\cdot\mathbf{v}])dS +\int_{V(t)}p(\nabla\cdot\mathbf{v})dV+\int_{V(t)}(\tau:\nabla\mathbf{v})dV \qquad (12)$$

The surface $S(t)$ consists of four parts:

- the fixed surfaces S_f, on which both $\mathbf{v}$ and $\mathbf{v}_S$ are zero
- the moving surfaces S_m, on which $\mathbf{v}=\mathbf{v}_S$, both being nonzero
- the cross-section of the entry port S_1, where $\mathbf{v}_S=0$
- the cross-section of the exit port S_2, where $\mathbf{v}_S=0$

Presently each of the surface integrals will be split into four parts corresponding to these four surfaces.

Going from Eq. (2) to Eq. (10), and from there to Eq. (12)

involves only mathematical manipulations. The next task is to interpret the terms in Eq. (12), and to introduce several assumptions.

The term on the left side can be interpreted as the time rate of change of the total kinetic and potential energies $(K_{tot}+\Phi_{tot})$ within the "control volume," whose shape and volume are changing with time. We next examine *seriatim* the five terms on the right side:

Term 1 contributes only at the entry and exit ports, with cross-sections of magnitude S_1 and S_2, and this gives the rate of influx and efflux of kinetic and potential energy:

$$\text{Term 1}=\left(\tfrac{1}{2}\rho_1\langle v_1^3\rangle S_1+\rho_1\hat{\Phi}_1\langle v_1\rangle S_1\right)-\left(\tfrac{1}{2}\rho_2\langle v_2^3\rangle S_2+\rho_2\hat{\Phi}_2\langle v_2\rangle S_2\right) \qquad (13)$$

The angular brackets indicate an average over the cross section. To get this result we have to assume that the fluid density and potential energy per unit mass are constant over the cross section, and that the fluid is flowing parallel to the tube walls at the entry and exit ports. The first term in Eq. (13) (which includes the minus sign in Eq. (12)) is positive, since at "1", $(\mathbf{n}\cdot\mathbf{v})=-v_1$, and the second term is negative, since at "2", $(\mathbf{n}\cdot\mathbf{v})=+v_2$.

Term 2 gives no contribution on S_f since $\mathbf{v}$ is zero there. On each surface element dS of S_m there is a force $\mathbf{n}pdS$ acting on a surface moving with a velocity $\mathbf{v}$, and the dot product of this quantity gives the rate at which the fluid does work on the element dS of the moving surface because of the pressure, and we use the symbol $W_m^{(p)}$ to indicate the integral over all these surface forces. Then the integrals over the surfaces S_1 and S_2 (including the minus sign in Eq. (12)) give the work required to push the fluid into the system at "1" and the work required to push the fluid out of the system at "2". Therefore Term 2 gives finally:

$$\text{Term 2}=p_1\langle v_1\rangle S_1-p_2\langle v_2\rangle S_2-W_m^{(p)} \qquad (14)$$

Here we have assumed that the pressure does not vary over the cross-section at the entry and exit ports.

Term 3 gives no contribution on S_f since $\mathbf{v}$ is zero there. The integral over S_m can be interpreted as the rate at which the fluid does

work on the moving surfaces by means of the viscous forces, and this integral is designated as $W_m^{(\tau)}$. At the entry and exit ports it is conventional to neglect the work associated with the viscous forces, since they are generally quite small compared with the pressure contributions. Therefore we get

$$\text{Term 3} = -W_m^{(\tau)} \tag{15}$$

Note that for both the pressure and viscous work terms, we do not include the minus sign in Eq. (12) in the definition of the quantities $W_m^{(p)}$ and $W_m^{(\tau)}$. We use the symbol $W_m = W_m^{(p)} + W_m^{(\tau)}$ to represent the total rate of doing work on the surroundings by means of the moving parts.

Terms 4 and 5 cannot be further simplified and we write

$$\text{Term 4} = +\int_{V(t)} p(\nabla\cdot\mathbf{v})dV = -E_c \tag{16}$$

$$\text{Term 5} = +\int_{V(t)} (\tau:\nabla\mathbf{v})dV = -E_v \tag{17}$$

For Newtonian fluids the quantity E_v is the rate at which mechanical energy is *irreversibly* degraded into thermal energy because of the viscosity of the fluid; it is always a positive quantity. Quite a lot is known about how to estimate E_v in a variety of flow situations [Ref. 7, §7.4]. (For viscoelastic fluids, which we are not considering here, E_v has to be interpreted differently and may even be negative.) The quantity E_c is the rate at which mechanical energy is *reversibly* changed into thermal energy because of the compressibility of the fluid, and it may be either positive or negative. If the fluid is assumed to be incompressible, then E_c is zero.

When all the contributions are inserted into Eq. (12) we finally obtain the MMEB:

$$\begin{aligned}\frac{d}{dt}(K_{\text{tot}}+\Phi_{\text{tot}}) &= \left(\tfrac{1}{2}\rho_1\langle v_1^3\rangle S_1 + \rho_1\hat{\Phi}_1\langle v_1\rangle S_1 + p_1\langle v_1\rangle S_1\right)\\ &-\left(\tfrac{1}{2}\rho_2\langle v_2^3\rangle S_2 + \rho_2\hat{\Phi}_2\langle v_2\rangle S_2 + p_2\langle v_2\rangle S_2\right) - W_m - E_c - E_v\end{aligned} \tag{18}$$

If, now, we introduce the symbols $w_1 = \rho_1\langle v_1\rangle S_1$ and $w_2 = \rho_2\langle v_2\rangle S_2$

for the mass rates of flow "in" and "out", then Eq. (18) can be rewritten as

$$\boxed{\frac{d}{dt}(K_{\text{tot}}+\Phi_{\text{tot}}) = -\Delta\left(\frac{1}{2}\frac{\langle v^3\rangle}{\langle v\rangle} + \hat{\Phi} + \frac{p}{\rho}\right)w - W_m - E_c - E_v} \tag{19}$$

This is the final form for the MMEB. It includes the time-dependent terms, the prescriptions for the averaging of the velocities, and explicit expressions for E_v and E_c (see Eqs. (16) and (17)). Keep in mind that several assumptions have been made in its derivation:

- the pressure, density, and potential energy do not vary over the cross sections at the entry and exit ports
- the fluid velocity is parallel to the tube walls at the entry and exit ports
- the stress tensor does not contribute to the work terms at the entry and exit ports

These are generally not serious limitations, but if situations arise where these assumptions are not valid, one can always go back to the general expressions in Eq. (12).

As long as there are moving parts in the system, there is no possibility for "steady state" flow. We can, however, talk about situations in which the total kinetic and potential energies are not changing with time and in which the mass rates of flow "in" and "out" are equal (most people would regard these as a "steady-state operations"). Then the MMEB becomes, if we use $\hat{\Phi} = gz$

$$\Delta\left(\frac{1}{2}\frac{\langle v^3\rangle}{\langle v\rangle} + gz + \frac{p}{\rho}\right) + \hat{W}_m + \hat{E}_c + \hat{E}_v = 0 \tag{20}$$

in which $\hat{W}_m = W_m/w$, and $\hat{E}_c$ and $\hat{E}_v$ are defined similarly. In the next section we attempt to go from Eq. (20) to Eq. (1).

§8. An alternative (approximate) version of the steady-state MMEB

If we assume that the velocity profiles are flat at the entry and exit planes, then $\langle v^3\rangle/\langle v\rangle$ is the same as v^2, and the kinetic energy terms in Eqs. (1) and (20) are reconciled. Relatively small errors are introduced by this assumption if the flow is turbulent.

It is more difficult to reconcile those terms in Eqs. (1) and (20) that involve the pressure. This requires getting an *approximate* expression for E_c defined in Eq. (16). We imagine that there is a representative streamline running through the system, and we introduce a coordinate s along the streamline as in §5 above. If there are moving parts and if the system geometry is quite complex, it may not be possible to do this. It is assumed that pressure, density, and velocity do not vary over the cross-section. We further imagine that at each position along the streamline, there is a cross-section $S(s)$ perpendicular to the s-coordinate, so that we can write $dV = S(s)ds$.

We start by using the fact that $(\nabla \cdot \rho\mathbf{v}) = 0$ at steady state so that

$$E_c = -\int_V p(\nabla \cdot \mathbf{v})dV = \int_V \frac{p}{\rho}(\mathbf{v} \cdot \nabla\rho)dV \tag{21}$$

Then we use the assumption that the pressure, density, and velocity are constant over the cross-section to write approximately

$$E_c = \int_1^2 \frac{p}{\rho}\left(v\frac{d\rho}{ds}\right)S(s)ds \tag{22}$$

Although ρ, v, and S are functions of the streamline coordinate s, their product, $w = \rho vS$, is a constant for steady-state operation and hence may be taken outside the integral. Therefore

$$E_c = w\int_1^2 \frac{p}{\rho^2}\left(\frac{d\rho}{ds}\right)ds = -w\int_1^2 p\frac{d}{ds}\left(\frac{1}{\rho}\right)ds \tag{23}$$

Integration by parts then gives

$$E_c = -w\left[\frac{p}{\rho}\Big|_1^2 - \int_1^2 \frac{1}{\rho}\frac{dp}{ds}ds\right] = -w\Delta\left(\frac{p}{\rho}\right) + w\int_1^2 \frac{1}{\rho}dp \tag{24}$$

When this result is put into Eq. (20) and flat velocity profiles are assumed, Eq. (1) is obtained. In Eq. (1) the integral term may be difficult to calculate, since the pressure-density relation has to be known along a "representative streamline." Similarly in Eq. (20) the $\hat{E}_c$ term may be difficult to evaluate, except when incompressible flow is assumed. Because of the questionable nature of the assumptions made in this section, it seems preferable to use Eq. (20) rather than Eq. (1). Also Eq. (20) is easily generalized to systems with multiple inlet and outlet ports, whereas Eq. (1) is not.

§9. Concluding comments regarding the MMEB

The derivation of Eq. (19) involves some mathematical manipulations, but there are really just two basic steps: (i) one takes the dot product of **v** with the equation of motion to generate the equation of change for mechanical energy, and (ii) the latter is integrated over the volume of the flow system. This generates the MMEB in its complete unsteady-state form. The equation of motion is based on the concept of balance of momentum, and this is the only concept from physics that is used.

The MMEB in Eq. (19) can be rewritten in terms of thermodynamic functions if one desires. In Ref. [7] the MMEB was written in terms of the Helmholtz and Gibbs free energies for isothermal flow in Eqs. 7.3-1 and 15.2-1, and in terms of internal energy and enthalpy for isentropic flow in Eq. 15.2-2. In retrospect, I wish we had not done this. Perhaps our writing of the MMEB in this form reinforced the misconception that thermodynamical arguments are needed in the derivation.

Let us next discuss the relation of the MMEB to other energy balances. The total energy balance can be obtained in two ways: it can be written directly by extending the first law of thermodynamics to an open system, or it can be obtained by integrating the equation of change for energy over the volume of the flow system by following the procedures in §7 above. By either method one gets the *macroscopic total energy balance*:

$$\frac{d}{dt}(K_{\text{tot}} + \Phi_{\text{tot}} + U_{\text{tot}}) = -\Delta\left(\frac{1}{2}\frac{\langle v^3\rangle}{\langle v\rangle} + \hat{\Phi} + \hat{U} + \frac{p}{\rho}\right)w + Q - W_m \tag{25}$$

According to this balance, the total energy (kinetic + potential + internal) changes with time for three reasons: (a) total energy enters and leaves the flow system through the ports and "1" and "2"; (b) heat is added to the system through the walls (the Q-term); and (c) the system does work on the surroundings, $W_m + p_2\langle v_2\rangle S_2 - p_1\langle v_1\rangle S_1$. When the MMEB of Eq. (19) is subtracted from Eq. (25) the following *macroscopic internal energy balance* is obtained:

$$\frac{d}{dt}U_{\text{tot}} = -\Delta(Uw) + Q + E_c + E_v \tag{26}$$

This interesting equation states that the internal energy of the system increases because of a difference in the rates of input and output of internal energy by flow, because of the rate of heat transfer to the system, and because of the rate of conversion of mechanical energy into thermal energy. I am unaware of any physical reasoning that leads to Eq. (26) directly; it is obtained by making use of the MMEB. It may also be obtained by integrating the equation of change for the internal energy over the volume of the flow system. However, that equation is derived by using the equation of motion (Ref. [7], §10.1).

Oblivious of this fact, some textbook authors write Eq. (26), or a simplified version thereof, and subtract it from the total energy balance in Eq. (25), and thereby obtain the MMEB of Eq. (19), with very little effort. This has apparently given rise to the incorrect notion that the MMEB is an "alternative form of the total energy balance." There are also textbook derivations that introduce the second law, but this is clearly not necessary.

From the above derivation it is clear that *neither the first nor the second law of thermodynamics is needed* to obtain the MMEB. For further comments on the misunderstandings regarding the MMEB, and some of the historical developments, the reader is referred to the article by Whitaker [10]. Also Astarita and Mackay have discussed the MMEB for viscoelastic fluids, but with the restriction to steady-state, incompressible flow [11].

So what should we tell our undergraduate students about the MMEB? We should certainly tell them what the physical basis of the equation is. It would not be inappropriate to give them a careful derivation of the classical Bernoulli equation and then give the plausibility arguments of §5 for adding on the work and dissipation terms; they should then be told, in words, what the complete derivation involves. Going through the entire development in §7 above would probably be all right for the best undergraduates, but it might be more than the average or below-average students can handle. But even they should be able to get the gist of the derivation. After all, it is just a two-step derivation: multiply the equation of motion by **v**, and then integrate! In any event, we should avoid giving them incorrect developments.

The macroscopic mass, momentum, energy, and angular momentum balances, along with the MMEB, should be presented to students as a set of equations to be used for describing macroscopic systems. It should be made clear that, although the macroscopic momentum balance and the MMEB both come from the

equation of motion, the two macroscopic balances contain *different* information; one is a vector equation and the other a scalar equation. In addition it should be emphasized that the use of the macroscopic balances usually requires some intuition about the system (based on experience), experimental information about the system, or flow visualization studies. Most textbooks (including [7]) have been remiss in not making this point clear.

The above discussion illustrates that careful thought is required for preparing lecture material and textbooks. Errors in textbooks tend to proliferate, and textbook authors must realize that an irresponsible comment may affect many generations of students. It is impossible to write error-free textbooks, of course, but to minimize the errors, textbook authors need a lot of uninterrupted time as well as conscientious and intelligent co-authors or assistants. Deans and department chairmen need to recognize the role of textbook preparation in upgrading undergraduate education and should give encouragement and assistance to those who are willing to engage in book-writing. Also those who have been imaginative and productive researchers should be given particular encouragement to assist in the preparation of the textbooks, monographs, and handbooks that will shape the next generation of engineers, teachers, and students.

The author will welcome any comments on his views about the MMEB and other matters discussed above. He can be reached at <bird@chewi.che.wisc.edu>.

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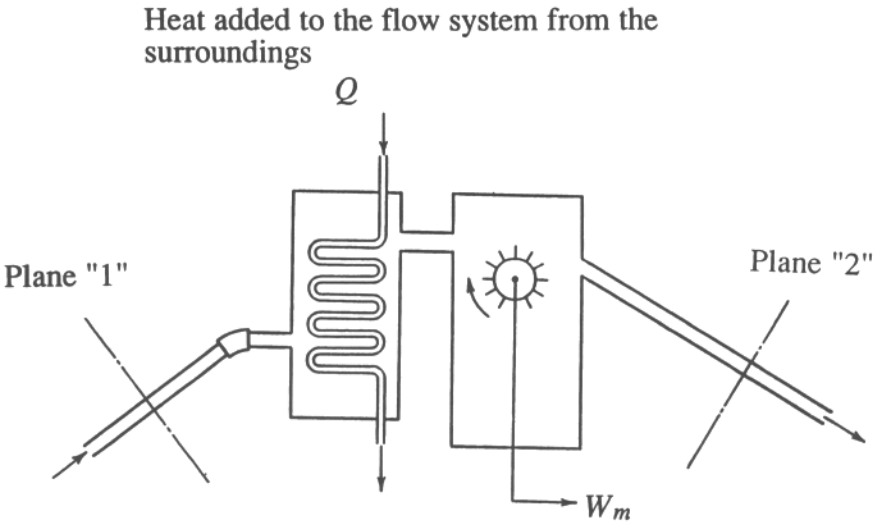

Figure 1. An engineering flow system operating at steady state.

Chemical Engineering at University of Missouri-Rolla

The University of Missouri School of Mines and Metallurgy, which in 1964 became the University of Missouri-Rolla, was founded in 1870 as the first technological institution west of the Mississippi River and one of the first in the nation. The new school was Missouri's response to the acute need for scientific and technical education in the developing nation and was a product of the Morrill Act of 1862.

The Department of Chemical Engineering at the University of Missouri-Rolla started as the Department of Chemical Engineering and Chemistry in 1915. The department was divided into the Department of Chemical Engineering and the Department of Chemistry in 1964, when the campus became part of the four-campus University of Missouri. Both are still housed in the same building and work closely together and both offer undergraduate and graduate degrees through the doctorate.

The University of Missouri-Rolla includes the School of Engineering, the School of Mines and Metallurgy, and the College of Arts and Sciences. The Department of Chemical Engineering is part of the School of Engineering. Total enrollment at UMR is about 5000 students, and in the Department of Chemical Engineering it is about 300 students beyond the freshman year. About 80 per cent of UMR students are engineering or science majors. The students benefit from working in a technological environment with well-equipped laboratories.